One algorithm from The Book*: A tribute to Ira Pohl*

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A9.com

http://www.stepanovpapers.com/IraPohlFest.pdf

The highest compliment [Erdős] could pay to a colleague's work was to say, “That's straight from The Book.”

Encyclopedia Britannica

CS needs its Book

The Book contains algorithms that are:

- Beautiful
- Optimal
- Useful

Programming Techniques

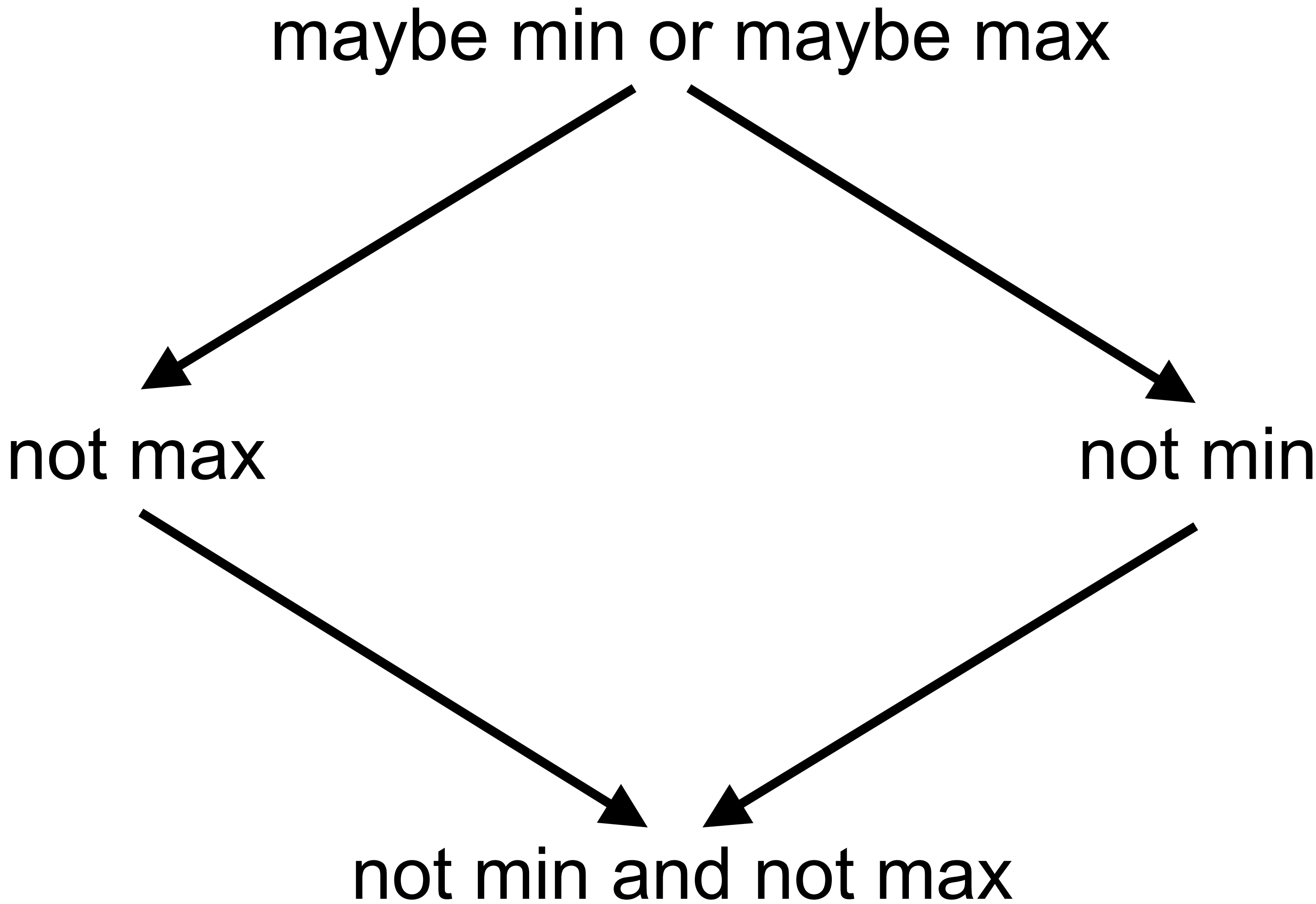
R. Morris
Editor

A Sorting Problem and Its Complexity

Ira Pohl
University of California*

Finding both *min* and *max*

- To find minimum (or maximum) of *n* elements we need *n – 1* comparisons
- Don't we need *2n – 2* (or *3?*) comparisons to find both?
- Ira showed that we need at most $\lceil \frac{3}{2}n \rceil - 2$ comparisons
- And he showed that his algorithm is optimal


maybe min or maybe max
not max
not min
not min and not max

Strict Weak Ordering

- Weak trichotomy

$$x \prec y \lor y \prec x \lor x \sim y$$

- Transitivity

$$(x \prec y \land y \prec z) \Rightarrow x \prec z$$

- Irreflexivity, or strictness

$$\neg(x \prec x)$$

```
template <StrictWeakOrdering R>
struct min_max
{
  R r;

  template <Regular T>  // T == Domain<R>
  const T& min(const T& x,
               const T& y) const {
    if (r(y, x)) return y;
    else         return x;
  }
```

Weak Commutativity

- Is min commutative?
- Not for StrictWeakOrdering
- Weak Commutativity!

$$a \circ b \sim b \circ a$$

- Set with min defined is
  - semigroup
  - (weak Abelian) semigroup
- Weak theories
  - equivalence axioms (instead of equational)

```
template <Regular T>  // T == Domain<R>
const T& max(const T& x,
             const T& y) const {
  if (r(y, x)) return x;
  else         return y;
}
```

```
// the idiot who designed STL wrote:
template <Regular T>  // T == Domain<R>
const T& max(const T& x,
             const T& y) const {
  if (r(x, y)) return y;
  else         return x;
}

// why is it wrong?
```

```
template <Regular T>  // T == Domain<R>
pair<T, T> construct(const T& x,
                     const T& y) const {
  if (r(y, x)) return {y, x};
  else         return {x, y};
}
```

```
  template <Regular T>  // T == Domain<R>
  pair<T, T>
  combine(const pair<T, T>& x,
          const pair<T, T>& y) const {
    return { min(x.first, y.first),
             max(x.second, y.second) };
  }
};
```

Iterators

- Input
- Forward
- Bidirectional
- RandomAccess

```
template <StrictWeakOrdering R>
struct compare_dereference
{
  R r;

  template <InputIterator I>
  // Domain<R> == ValueType<I>
  bool operator()(const I& i,
                  const I& j) const {
    return r(*i, *j);
  }
};
```

```
template <ForwardIterator I,
          StrictWeakOrdering R>
pair<I, I>
min_max_element_even_length(I first,
                            I last,
                            R r) {
  // assert(distance(first, last) % 2 == 0)
  min_max<compare_dereference<R>> op{r};
  if (first == last) return {last, last};
```

```
  I prev = first;
  pair<I, I> result =
                    op.construct(prev, ++first);
  while (++first != last) {
    prev = first;
    result = op.combine(
                    result,
                    op.construct(prev, ++first));
  }
  return result;
}
```

```
template <ForwardIterator I,
          StrictWeakOrdering R>
pair<I, I>
min_max_element(I first, I last, R r) {
  min_max<compare_dereference<R>> op{r};
  I prev = first;
  if (first == last || ++first == last)
     return {prev, prev};
```

```
  pair<I, I> result =
                    op.construct(prev, first);
  while (++first != last) {
    prev = first;
    if (++first == last)
      return op.combine(result,
                             {prev, prev});
    result = op.combine(
                    result,
                    op.construct(prev, first));
  }
  return result;
}
```

Type Functions

```
template <InputIterator I>
using ValueType = typename
      std::iterator_traits<I>::value_type;
```

```
template <InputIterator I,
          StrictWeakOrdering R>
pair<ValueType<I>, ValueType<I>>
min_max_value_nonempty(I first,
                       I last,
                       R r) {
  typedef ValueType<I> T;
  min_max<R> op{r};
  T val = *first;
  if (++first == last) return {val, val};
```

```
  pair<T, T> result =
                  op.construct(val, *first);
  while (++first != last) {
    val = *first;
    if (++first == last)
      return op.combine(result,
                        {val, val});
    result = op.combine(
                  result,
                  op.construct(val, *first));
  }
  return result;
}
```

```
template <InputIterator I,
          StrictWeakOrdering R>
pair<ValueType<I>, ValueType<I>>
min_max_value(I first, I last, R r) {
  typedef ValueType<I> T;
  if (first == last)
    return {supremum(r), infimum(r)}
  return min_max_value_nonempty(first,
                                last,
                                r);
}
```

- I have been teaching this algorithm every 2 – 3 years for the last 30 years
- When I teach it, I implement it anew
- Writing the code and teaching it gives me joy every time

THANK YOU, IRA!

Getting rid of an extra compare

```
// Add to min_max:
template <Regular T>  // T == Domain<R>
pair<T, T> combine(const pair<T, T>& x,
                   const T& val) const {
  if (r(val, x.first)) return { val, x.second};
  if (r(val, x.second)) return x;
  return {x.first, val};
}
```

Getting rid of an extra compare (2)

```
// In min_max_element and
// min_max_value_nonempty, replace:
if (++first == last)
  return op.combine(result, {val, val});

// with
if (++first == last)
  return op.combine(result, val);
```